CAREER AVENUES

Aim for excellence in **Engineering**

DIGITAL CIRCUITS IN

Digital Circuits

Basic Scope and Introduction

This book covers theory solved examples and previous year gate question for following topics:

Number system, Boolean algebra, logic gates, digital IC families (DTL, TTL, ECL, MOS, CMOS).

Combinatorial circuits: arithmetic circuits, code converters, multiplexers, decoders, PROMs and PLAs.

Sequential circuits: latches and flip-flops, counters and shift-registers. Sample and hold circuits, ADCs, DACs, Semiconductor memories.

Microprocessor (8085): architecture, programming, memory and I/O interfacing.

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CHAPTER 1- NUMBER SYSTEMS

I. INTRODUCTION

DECIMAL NUMBERS

The position of each digit in a weighted number system is assigned a weight based on the **base** or **radix** of the system. The **radix** of decimal numbers is **ten**, because only ten symbols (0 through 9) are used to represent any number.

The column weights of decimal numbers are **powers of ten** that increase from right to left beginning with $10^0 = 1$:

$$\ldots 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.$$

For **fractional** decimal numbers, the column weights are **negative powers** of ten that decrease from left to right:

$$10^2\ 10^1\ 10^0.\ 10^{-1}\ 10^{-2}\ 10^{-3}\ 10^{-4}\ \ldots$$

Decimal numbers can be expressed as the **sum of the products** of each digit times the column value for that digit. Thus, the number **9240** can be expressed as:

$$(\mathbf{9} \times 10^3) + (\mathbf{2} \times 10^2) + (\mathbf{4} \times 10^1) + (\mathbf{0} \times 10^0)$$

or

$$\mathbf{9} \times 1{,}000 + \mathbf{2} \times 100 + \mathbf{4} \times 10 + \mathbf{0} \times 1$$

Example: Express the number **480.52** as the sum of values of each digit.
Solution: $\mathbf{480.52} = (\mathbf{4} \times 10^2) + (\mathbf{8} \times 10^1) + (\mathbf{0} \times 10^0) + (\mathbf{5} \times 10^{-1}) + (\mathbf{2} \times 10^{-2})$

BINARY NUMBERS

For digital systems, the binary number system is used. Binary has a **radix** of **two** and uses the digits **0** and **1** to represent quantities. The column weights of binary numbers are **powers of two** that increase from right to left beginning with $2^0 = 1$:

$$\mathbf{\ldots 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0.}$$

For **fractional** binary numbers, the column weights are **negative powers** of two that decrease from left to right:

$$\mathbf{2^2\ 2^1\ 2^0.\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ \ldots}$$

HEXADECIMAL NUMBERS

- Base or radix is 16 for Hexadecimal number system.
- 1 hex digit is equivalent to 4 binary bits.
- Numbers are 0, 1, 2…..8, 9, A, B, C, D, E, and F.
- Numbers are expressed as powers of 16.
- Column weights $16^4 = 65536$, $16^3 = 4096$, $16^2 = 256$, $16^1 = 16$, $16^0 = 1$
- For fractional hex number $16^2\ 16^1\ 16^0.\ 16^{-1}\ 16^{-2}\ 16^{-3}\ 16^{-4}\ \ldots$

OCTAL NUMBERS

- Base or radix is 8 for Octal number system.
- 1 octal digit is equivalent to 3 binary bits.
- Octal numbers are 0 to7.
- Numbers are expressed as powers of 8.
- Column weights **$8^{3=}$512, 8^2 =64, 8^1 =8, 8^0 =1**
- For fractional hex number 8^2 8^1 8^0. 8^{-1} 8^{-2} 8^{-3} 8^{-4} …

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

II. NUMBER SYSTEM CONVERSIONS

BINARY -TO-DECIMAL CONVERSIONS (base 2 to base 10)

The decimal equivalent of a binary number can be determined by adding the column values of all of the bits that are **1** and discarding all of the bits that are 0.

Example: Convert the binary number **100101.01** to decimal.
Solution: Start by writing the column weights; then add the weights that correspond to each **1** in the number.

2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}		
32	16	8	4	2	1	.	½	¼		
1	0	0	1	0	1	.	0	1		
32			+4		+1			+¼	=	37¼

Example: convert $(1000100)_2$ to decimal

= 64 + 0 + 0+ 0 + 4 + 0 + 0
= $(68)_{10}$

DECIMAL-TO-BINARY CONVERSIONS (base 10 to base 2)

You can convert a decimal whole number to binary by reversing the procedure. Write the decimal weight of each column and place **1**'s in the columns that sum to the decimal number.

Example: Convert the decimal number **49** to binary.

Solution: Write down column weights until the last number is **larger** than the one you want to convert.

2^6	2^5	2^4	2^3	2^2	2^1	2^0.
64	**32**	**16**	**8**	**4**	**2**	**1.**
0	**1**	**1**	**0**	**0**	**0**	**1.**

Instead of division, multiplication by 2 is carried out and the integer part of the result is saved and Placed after the decimal point. The fractional part is again multiplied by 2 and the process repeated.

Example: convert $(68)_{10}$ to binary

Solution:

68/2 = 34 remainder is 0
34/ 2 = 17 remainder is 0
17 / 2 = 8 remainder is 1
8 / 2 = 4 remainder is 0
4 / 2 = 2 remainder is 0
2 / 2 = 1 remainder is 0
1 / 2 = 0 remainder is 1
Answer = 1 0 0 0 1 0 0
Note: the answer is read from bottom (MSB) to top (LSB) as 10001002

DECIMAL FRACTION-TO-BINARY FRACTION CONVERSIONS

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

Example: Convert the decimal fraction **0.188** to binary by repeatedly **multiplying** the fractional results by 2.

Solution:

0.188 x 2 = 0.376 **carry = 0** **MSB**
0.376 **x 2 = 0**.752 **carry = 0**
0.752 **x 2 = 1**.504 **carry = 1**
0.504 **x 2 = 1**.008 **carry = 1**
0.008 **x 2 = 0**.016 **carry = 0** **LSB**
Answer = .00110 (for **five** significant digits**)**

Example: convert (0.68)10 to binary fraction.

Solution:

0.68 * 2 = 1.36 integer part is 1
0.36 * 2 = 0.72 integer part is 0
0.72 * 2 = 1.44 integer part is 1
0.44 * 2 = 0.88 integer part is 0
Answer = 0. 1 0 1 0…..

Example: convert (68.68)10 to binary equivalent.
Answer = 1 0 0 0 1 0 0 . 1 0 1 0….

CONVERTING BINARY TO HEX AND OCTAL

Conversion of binary numbers to octal and hex simply requires grouping bits in the binary numbers into groups of three bits for conversion to octal and into groups of four bits for conversion to hex.
Groups are formed beginning with the LSB and progressing to the MSB.

Thus, $\underline{11}\ \underline{100}\ \underline{111}_2 = 347_8$ $\underline{1110}\ \underline{0111}_2 = E7_{16}$
$\underline{11}\ \underline{100}\ \underline{010}\ \underline{101}\ \underline{010}\ \underline{010}\ \underline{001}_2 = 3025221_8$ $\underline{1}\ \underline{1000}\ \underline{1010}\ \underline{1000}\ \underline{0111}_2 = 18A87_{16}$

CONVERTING HEX TO DECIMAL (base 16 to base 10)

Example: Express $\mathbf{1A2F_{16}}$ in decimal and binary.
Solution: Start by writing the column weights:

4096	**256**	**16**	**1**
1	**A**	**2**	**F**

1(4096) + 10(256) +2(16) +15(1) = $\mathbf{6703_{10}}$

Example: convert $(F4C)_{16}$ to decimal

= (F x 162) + (4 x 161) + (C x 160)
= (15 x 256) + (4 x 16) + (12 x 1)

CONVERTING OCTAL TO DECIMAL (base 8 to base 10)

Example: Express $\mathbf{3702_8}$ in decimal
Solution: Start by writing the column weights:

512	64	8	1
3	**7**	**0**	**2**

3(512) + 7(64) +0(8) +2(1) = $\mathbf{1986_{10}}$

CONVERTING DECIMAL TO HEX (base 10 to base 16)

Example: convert $(4768)_{10}$ to hex.
Solution: Start by dividing with 16
= 4768 / 16 = 298 remainder 0
= 298 / 16 = 18 remainder 10 (A)
= 18 / 16 = 1 remainder 2
= 1 / 16 = 0 remainder 1
Answer: 1 2 A 0 = $(12A0)_{16}$
Note: the answer is read from bottom to top, same as with the binary case.
= 3840 + 64 + 12 + 0
= (3916)10

CONVERTING DECIMAL TO OCTAL (base 10 to base 8)
Example: convert $(177)_{10}$ to octal
Solution: Start by dividing with 16

177 / 8 = 22 remainder is 1
22 / 8 = 2 remainder is 6
2 / 8 = 0 remainder is 2
Answer = 2 6 1 = $(261)_8$

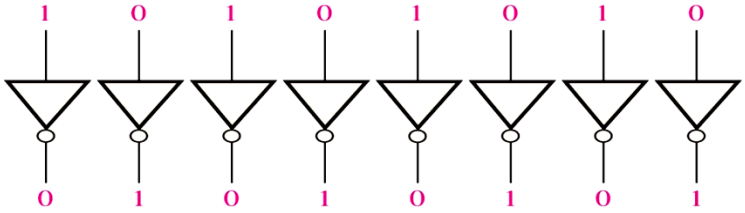
Note: the answer is read from bottom to top as $(261)_8$, the same as with the binary case.

Conversion of decimal fraction to octal fraction is carried out in the same manner as decimal to binary except that now the multiplication is carried out by 8. To convert octal to hex first convert octal to binary and then convert binary to hex. To convert hex to octal first convert hex to binary and then convert binary to octal.

III. COMPLEMENTS OF BINARY NUMBERS

1'S COMPLEMENT

The 1's complement of a binary number is just the inverse of the digits. To form the 1's complement, change all **0**'s to **1**'s and all **1**'s to **0**'s.
For example, the 1's complement of **11001010** is 00110101
In digital circuits, the 1's complement is formed by using inverters:



2'S COMPLEMENT:

The **2**'s complement of a binary number is found by adding **1** to the **LSB** of the **1**'s complement.

Recall that the 1's complement of **11001010** is
00110101 (**1**'s complement)
To form the 2's complement, add 1: +**1**
00110110 (2's complement)

Tip: To find 2's compliment directly copy LSB to answer if it is 0 copy next LSB and so on till you get 1. After copying 1 do 1's compliment of rest of MSB

GENERAL "B'S COMPLEMENT" NUMBERS FOR BASE B AND BASE 2

Everything flows in general the same as above except for the changes shown in the table below. In this table, "b" in column 2 represents a general base b number. Column 3 represents binary numbers.
Row 2 gives the representation of the number -1 (with leading digits equal to b-1, b the base).

Row 3 gives the equation for complementing each of the digits (in each case, subtract the digit from b-1, b the base)
Row 4 gives examples of positive numbers (with leading zeros extending leftward to infinity).
Row 5 shows the complement of the number in row 4 (and shows that the sum of the two equals zero)

	Base 10's (base - 1) = 10 - 1 = 9	**Base b (base - 1) = (b - 1) = B**	**Base 2 (base - 1) = (2 - 1) = 1**
Number = -1	…9…99999999999	…B…BBBBBBBBBB	…1…11111111111
Complement d_k of digit c_k	$d_k = 9 - c_k$	$d_k = (b-1) - c_k$ $= B - c_k$	$d_k = 1 - c_k$ If $c_k = 1,\ d_k = 0$ If $c_k = 0,\ d_k = 1$
Example of 8 digit positive number (leading 0 required)	00034857	For base 5, as an example, 00340240	01011010
Example of 8 digit negative number (leading (b-1) required) with magnitude equal to positive number in row above	00034857 -> 99965143 00034857 + 99965143 00000000	00340240 -> 44104210 00340240 + 44104210 00000000	01011010 -> 10100110 01011010 + 10100110 00000000

IV. ARITHMETIC FOR VARIOUS NUMBER SYSTEM

BINARY ADDITION

The **rules** for binary addition are:

0 + 0 = 0 Sum = 0, carry = 0
0 + 1 = 1 Sum = 1, carry = 0
1 + 0 = 1 Sum = 1, carry = 0
1 + 1 = 10 Sum = 0, carry = 1

When an input (carry = 1) due to a previous result, the **rule** is:

1 + 1 + 1 = 11 Sum = 1, carry = 1

Example: Add the binary numbers **00111** and **10101** and show the equivalent decimal addition

Solution:

111	
00111	**7**
10101	**21**
11100	**28**

BINARY SUBTRACTION

The **rules** for binary subtraction are:

0 - 0 = 0
1 - 1 = 0
1 - 0 = 1
10 - 1 = 1 with a borrow of **1**

Example: Subtract the binary number **00111** from **10101** and show the equivalent decimal subtraction.

Solution:

```
  111
 10101   21
-00111    7
 01110   14
```

HEXADECIMAL ADDITION

Examples

Carry	0	00	00	
Addend	3B2	3B2	3B2	3B2
Augend	+ 41C	+ 41C	+ 41C	+ 41C
Sum		E	CE	7CE

Carry	1	1	1	1	
Addend	A27	A27	A27	A27	A27
Augend	+ C3B	+ C3B	+ C3B	+ C3B	+ C3B
Sum	2	62	662	1662	

HEXADECIMAL SUBTRACTION

Uses the same principle of "borrowing" that decimal and binary subtraction uses.

Example

borrow				
Minuend	6E	6E	6E	
Subtrahend	- 29	- 29	-29	
Difference		5	45	

Minuend	AC3	AC3	AC3	AC3
Subtrahend	- 604	- 604	- 604	- 604
Difference		F	BF	4BF

OCTAL ADDITION

Examples

carry		1		
Addend	127	127	127	127
Augend	+ 42	+ 42	+ 42	+ 42
Sum		1	71	171

carry		1	11	11	1
Addend	1777	1777	1777	1777	1777
Augend	+ 777	+ 777	+ 777	+ 777	+ 777
Sum		6	76	776	2776

OCTAL SUBTRACTION

This is performed exactly like binary and decimal subtraction with the borrowing technique. Whenever the subtrahend is larger than the minuend, a 1 is borrowed from the next column.

Example

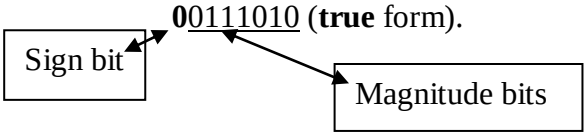
Minuend	124	124	124
Subtrahend	- 63	- 63	-63
Sum		1	41

V. SIGNED BINARY NUMBERS

There are several ways to represent signed binary numbers. In all cases, the **MSB** in a signed number is the **sign bit**, that tells you if the number is **positive** or **negative**.

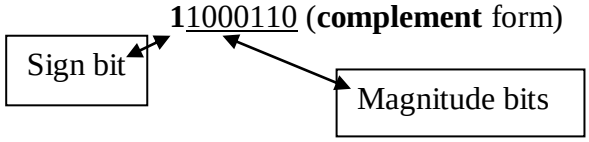
Computers use a **modified 2**'s complement for signed numbers. **Positive** numbers are stored in ***true*** form (with a 0 for the sign bit) and **negative** numbers are stored in *complement* form (with a **1** for the sign bit).

For example, the **positive** number **58** is written using **8**-bits in **true** (**un-complemented**) as:



Negative numbers are written as the **2**'s complement of the corresponding positive number.

The negative number -58 is written as:



An easy way to read a signed number that uses this notation is to assign the **sign bit** a column **weight** of **-128** (for an **8**-bit number). Then add the column weights for the **1**'s.

Example: Assuming that the sign bit = -128, show that **11000110** = **-58** as a **2**'s complement signed number.

Solution: Column weights -128 64 32 16 8 4 2 1.

1 1 0 0 0 1 1 0

-128 +64 +4 +2 = -58

ARITHMETIC OPERATIONS WITH SIGNED NUMBERS

ADDITION

Using the **signe**d number notation with **negative** numbers in **2**'s complement form simplifies addition and subtraction of signed numbers.

Rules for addition: Add the two **signed** numbers. **Discard** any **final carries**. The result is in **signed** form.

Examples:

00011110 = +30	**00001110 = +14**	**11111111 = -1**
00001111 = +15	**11101111 = -17**	**11111000 = -8**
00101101 = 45	**11111101 = -3**	**1 11110111 = -9**

Discard Final

OVERFLOW CONDITION

When two numbers are added and the **number of bits** required to represent the sum **exceeds** the number of bits in the two numbers, an **overflow** results as indicated by an **incorrect sign bit**. An overflow can occur only when both numbers have the same sign.

Two examples are:

01000000 = +128	Discard	**10000001 = -127**
01000001 = +129		**10000001 = -127**
10000001 = -126		**1 00000010 = -2**

Wrong! The answer is incorrect and the sign bit has changed

SUBTRACTION

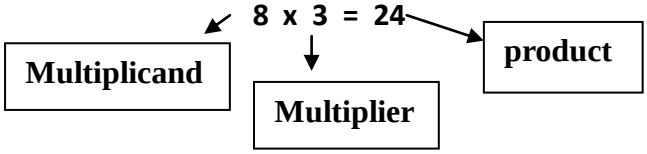
Rules for subtraction: **2**'s complement the subtrahend and **add** the numbers. **Discard** any **final carries**. The result is in **signed** form.

Examples:

00011110 = +30	**11111111 = -1**
11110001 = -15	**00001000 = +8**
1 00001111 = +15	**1 00000111 = +7**
Discard Final	Discard Final

MULTIPLICATION

The numbers in a multiplication are the **multiplicand**, the **multiplier**, and the **product**. These are illustrated in the following decimal multiplication:



The **multiplication** operation in most computers is accomplished using **addition** in two different ways**: direct addition** and **partial products**.

In direct addition method, you add the multiplicand a number of times equal to the multiplier. The disadvantage of this approach is that it becomes very lengthy if the multiplier is a large number. For example, to multiply **350** x **75**, you must add 350 to itself **75** times.

The **partial product** method is more common, the multiplicand is multiplied by each multiplier digit begins with the least significant digit, and the result is called a **partial product.** Each successive partial product is moved (**shifted**) one place to the left, and when all partial products have been produced, they are **added** to get the **final** product.

The **rules** for partial product are:

Determine if the **sign** of the **multiplicand** and **multiplier** are the same or different:

If the signs are the same, the product is positive

If the signs are **different**, the product is **negative**

Change any **negative** number to **true** form (uncomplemented).

Starting with the LSB of the multiplier, generate partial products, shift each successive partial product one bit to the left, and add each successive partial product to the sum of the previous partial products to get the **final** product.

If the sign bit that was determined in step 1 is negative, take the 2' complement of the product. If positive, leave the product in true form

Attach the sign bit to the product.

Example: Multiply the **signed** binary numbers: **0**1010011 (multiplicand) and **1**1000101 (multiplier).

Solution:

Since the two numbers have different signs, the sign bit of the product will be negative (**1**).

Take the **2'complement** of the multiplier to put it in **true** form **1**1000101 **--> 0**0111011

The multiplication proceeds as follows. Notice that only the **magnitude** bits are used

1. Since the final product should be **negative**, take the **2'complemrnt** of the final product
   1001100100001 0110011011111
2. Attach the sign bit → **1**0110011011111

	1010011	**Multiplicand**
x	**0**111**0**11	**Multiplier**
	1010**1**1	1st partial product
+	1010011	2nd partial product
	11111**0**01	**Sum** of 1st and 2nd
+	**0000000**	3rd partial product
	01111**1**001	**Sum**
+	1010011	4th partial product
	1110**1**0001	**Sum**
+	1010011	5th partial product
	100011**0**00001	**Sum**
+	1010011	6th partial product
	100110**0**100001	**Sum**
+	**0000000**	7th partial product
	1001100100001	**Final (Sum) product**

DIVISION

The numbers in a division are the **dividend**, the **divisor**, and the **quotient**. These are illustrated in the following standard format:

dividend / divisor = quotient

The **division** operation in computers is accomplished using **subtraction**. Since subtraction is done with an **adder**, division can also be accomplished with **adder**.

The result of a division is called the **quotient**; the quotient is the number of times that the divisor will go into the dividend. This means that the divisor can be **subtracted** from the dividend a number of times equal to the **quotient**, as illustrated by dividing **21** by **7** using subtraction operations:

21 – **7** = 14 – **7** = 7 – **7** = 0

In this simple example, the divisor was subtracted from the dividend **3** times before a **reminder** of **0** was obtained. Therefore, the quotient is **3**.

When two binary numbers are divided, both numbers must be in **true** (uncomplemented) form. The basic **rules** in a division process are as follows:

1. Determine if the **sign** of the **dividend** and **divisor** are the same or different:
   - If the signs are **the same**, the **quotient** is **positive**
   - If the sign are **different**, the **quotient** is **negative**
2. Subtract the **divisor** from the **dividend** using **2's complement addition** to get the first partial remainder and add **1** to the quotient. If the partial **reminder** is **positive**, go to step to step **3**. If the partial **reminder** is zero or negative, the division is complete.
3. Subtract the divisor from the partial reminder and add **1** to the quotient. If the result is positive, repeat for the next partial reminder. If the result is zero or negative, the division is complete.

4. Continue to **subtract** the divisor from the dividend and the partial reminder until there is a **zero** or **negative** result.
5. Count the number of times that the divisor is subtracted and you have the quotient.

Example: Divide 01100100 by 00011001
Solution:

1. The sign of both numbers are positive, so the quotient will be positive. The quotient is initially zero: **00000000**.

2. Subtract the divisor from the dividend using 2's complement addition (remember that the final carries are discarded).

```
  01100100      dividend
+ 11100111      2'complement of divisor
  01001011      positive 1st partial reminder
```

Add 1 to quotient: 00000000 + 00000001 = 00000001

3. Subtract the divisor from the 1st partial reminder using 2's complement addition.

```
  01001011      1st partial reminder
+ 11100111      2'complement of divisor
  00110010      positive 2nd partial reminder
```

Add 1 to quotient: 00000001 + 00000001 = 00000010

4. Subtract the divisor from the 2nd partial reminder using 2's complement addition.

```
  00110010      2st partial reminder
+ 11100111      2'complement of divisor
  00011001      positive 3rd partial reminder
```

Add 1 to quotient: 00000010 + 00000001 = 00000011

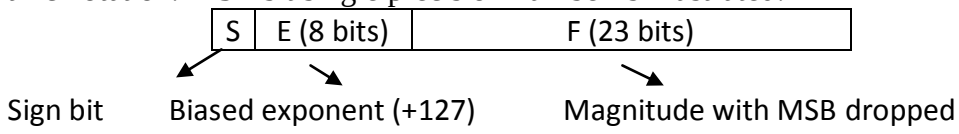
5. Subtract the divisor from the 3rd partial reminder using 2's complement addition.

```
  00011001      3rd partial reminder
+ 11100111      2'complement of divisor
  00000000      zero reminder
```

Add 1 to quotient: 00000011 + 00000001 = 00000100 (final quotient), the process is complete.

VI. FLOATING POINT NUMBERS

Floating point notation is capable of representing **very large** or **small** numbers by using a form of scientific notation. A 32-bit single precision number is illustrated:



Example: Express the speed of light, ***c***, in single precision floating point notation.
($c = 0.2998 \times 10^9$)

Solution:

Binary: c = 0001 0001 1101 1110 1001 0101 1100 0000_2

Scientific notation: c = 1.001 1101 1110 1001 0101 1100 0000 x 2^{28}

Floating point notation:

0	10011011	001 1101 1110 1001 0101 1100

S = 0 because the number is positive.

E = 28 + 127 = 155_{10} = 1001 1011_2.

F = the next 23 bits after the first 1 is dropped

VII. BINARY CODES

BINARY CODED DECIMAL (BCD)

Binary coded decimal (**BCD**) is a weighted code that is commonly used in digital systems when it is necessary to show decimal numbers such as in clock displays.

The table illustrates the difference between straight binary and BCD. **BCD** represents each decimal digit with a **4-bit** code. Notice that the codes 1010 through 1111 are not used in BCD.

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001

GRAY CODE

Gray code is an **unweighted** code that has a **single bit change** between one code word and the next in a sequence. Gray code is used to avoid problems in systems where an error can occur if more than one bit changes at a time.

A shaft encoder is a typical application. Three IR emitter/detectors are used to encode the position of the shaft. The encoder on the left uses binary and can have three bits change together, creating a potential error. The encoder on the right uses gray code and only 1-bit changes, eliminating potential errors.

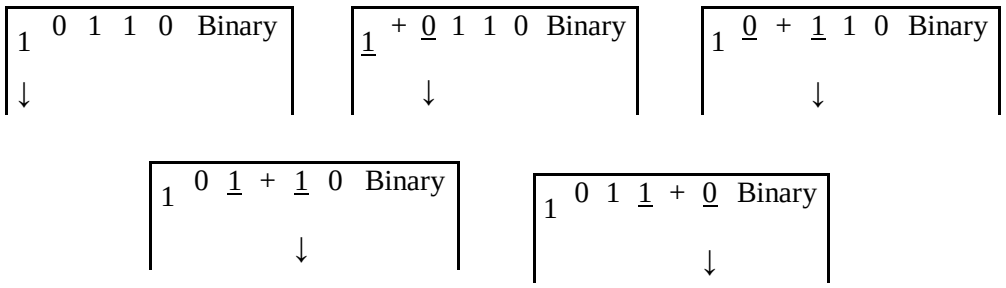
Decimal	Binary	Gray
0	000	0000
1	001	0001
2	010	0011
3	011	0010
4	100	0110
5	101	0111
6	110	0101
7	0111	0100

Binary-to-Gray Conversion

The algorithm to convert a binary value to its corresponding standard Gray code value is as follows:

1. Retain the MSB.
2. From left to right, add each adjacent pair of binary code bits to get the next Gray code bit, discarding the carry.

The following example shows the conversion of binary number $(10110)_2$ to its corresponding standard Gray code value, $(11101)_{Gray}$.

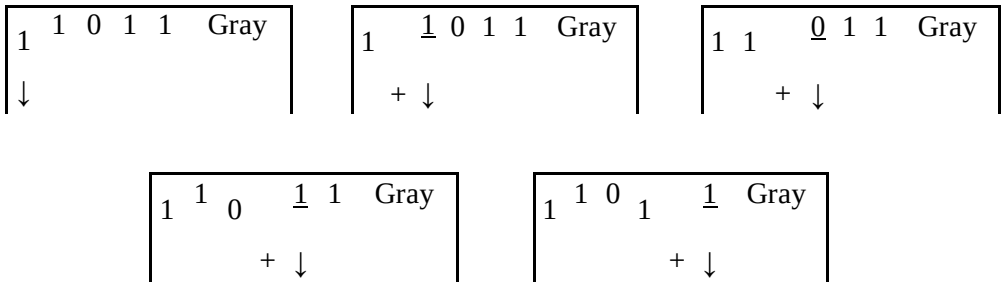


Gray-to-Binary Conversion

The algorithm to convert a standard Gray code value to its corresponding binary value is as follows:

1. Retain the MSB.
2. From left to right, add each binary code bit generated to the Gray code bit in the next position, discarding the carry.

The following example shows the conversion of the standard Gray code value $(11011)_{Gray}$ to its corresponding binary value, $(10010)_2$.



Excess-3 Code

The *Excess-3 code* uses a bias value of three. It means we have to add 3(0011) to the corresponding BCD code. Hence the codes for the ten digits 0, 1, …, 9 are 0011, 0100, …, 1100 respectively. The decimal number 294 would be represented as 0101 1100 0111.

84-2-1 Code

The *84-2-1 code* uses the weights of 8, 4, -2 and -1 in the coding. The decimal number 294 would be represented as 0110 1111 0100.

2421 Code

The *2421 code* uses the weights of 2, 4, 2 and 1 in the coding. According to the weights, certain digits may have alternative codes. For instance, the digit 3 could be represented as 0011 or 1001. However, we pick the former in the standard 2421 coding, so that the codes for the first five

digits 0 – 4 begin with 0, whereas the codes for the last five digits 5 – 9 begin with 1. The decimal number 294 would be represented as 0010 1111 0100.

ASCII CODE

ASCII is a code for alphanumeric characters and control characters. In its original form, **ASCII** encoded **128** characters and symbols using **7-bits**. The first **32** characters are control characters, that are based on obsolete teletype requirements, so these characters are generally assigned to other functions in modern usage**.**

In 1981, IBM introduced extended **ASCII**, which is an **8-bit** code and increased the character set to **256**. Other extended sets (such as **Unicode**) have been introduced to handle characters in languages other than English.

ERROR DETECTION

- Transmitted binary information is subject to noise that could change bits 1 to 0 and vice versa
- An *error detection code* is a binary code that detects digital errors during transmission
- The detected errors cannot be corrected, but can prompt the data to be retransmitted
- The most common error detection code used is the *parity bit*
- A parity bit is an extra bit included with a binary message to make the total number of 1's either odd or even

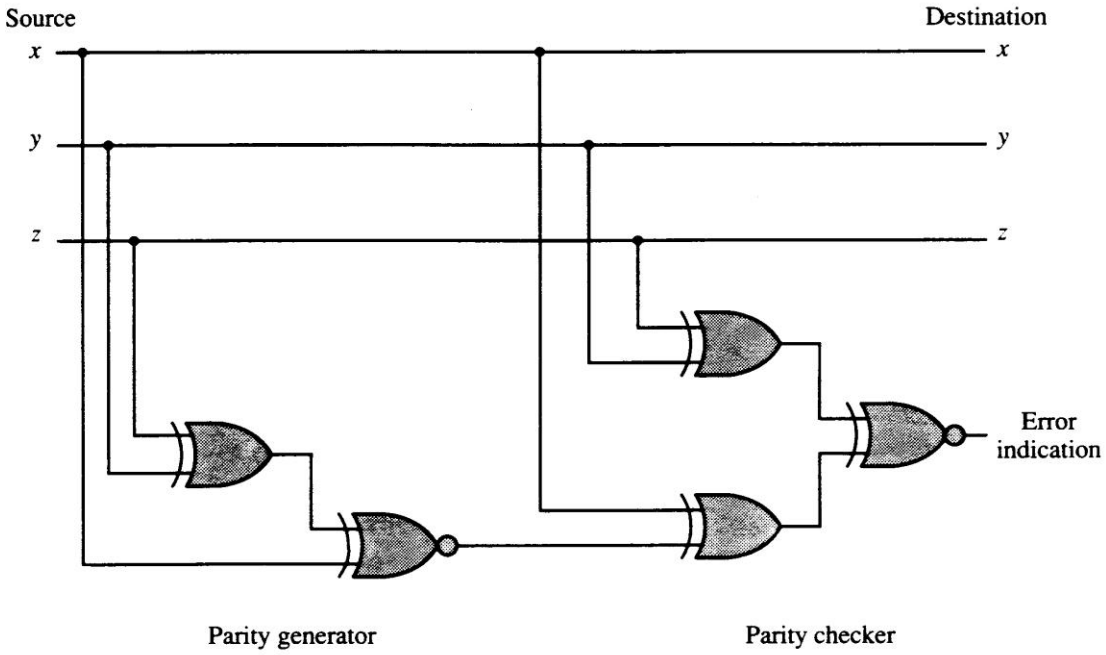
TABLE 3-7 Parity Bit Generation

Message *xyz*	*P*(odd)	*P*(even)
000	1	0
001	0	1
010	0	1
011	1	0
100	0	1
101	1	0
110	1	0
111	0	1

- The P(odd) bit is chosen to make the sum of 1's in all four bits odd
- The even-parity scheme has the disadvantage of having a bit combination of all 0's
- Procedure during transmission:
    - At the sending end, the message is applied to a *parity generator*
    - The message, including the parity bit, is transmitted
    - At the receiving end, all the incoming bits are applied to a *parity checker*
    - Any odd number of errors are detected

- Parity generators and checkers are constructed with XOR gates (odd function)
- An odd function generates 1 iff an odd number if input variables are 1

Figure 3-3 Error detection with odd parity bit.



Quick Review Questions

2-1. Convert the binary number 1011011 to its decimal equivalent.
a. 5 b. 63 c. 91 d. 92 e. 139

2-2. What is the weight of the digit '3' in the base-7 number 12345?
a. 3 b. 7 c. 14 d. 21 e. 49

2-3. Which of the following has the largest value?
a. $(110)_{10}$ b. $(10011011)_2$ c. $(1111)_5$ d. $(9A)_{16}$ e. $(222)_8$

2-4. If $(321)_4 = (57)_{10}$, what is the decimal equivalent of $(321000000)_4$?
a. 57×10^4 b. 57×10^6 c. 57×4^4 d. 57×4^6 e. 57^4

2-5. Convert each of the following decimal numbers to binary (base two) with at most eight digits in the fractional part, rounded to eight places.
a. 2000 b. 0.875 c. 0.07 d. 12.345

2-6. Convert each of the decimal numbers in Question 2-5 above to septimal (base seven) with at most six digits in the fractional part, rounded to six places.

2-7. Convert each of the decimal numbers in Question 2-5 above to octal (base eight) with at most four digits in the fractional part, rounded to four places.

2-8. Convert each of the decimal numbers in Question 2-5 above to hexadecimal (base sixteen) with at most two digits in the fractional part, rounded to two places.

2-9. Which of the following octal values is equivalent to the binary number $(110001)_2$?
a. $(15)_8$ b. $(31)_8$ c. $(33)_8$ d. $(49)_8$ e. $(61)_8$

2-10. Convert the binary number $(1001101)_2$ to
a. quaternary b. octal c. decimal d. hexadecimal

2-11. What is $(1011)_2 \times (101)_2$?
a. $(10000)_2$ b. $(110111)_2$ c. $(111111)_2$ d. $(111011)_2$ e. $(101101)_2$

2-12. Perform the following operations on binary numbers.
a. $(10111110)_2 + (10001101)_2$
b. $(11010010)_2 - (01101101)_2$
c. $(11100101)_2 - (00101110)_2$

2-13. In a 6-bit 2's complement binary number system, what is the decimal value represented by $(100100)_{2s}$?
a. –4 b. 36 c. –36 d. –27 e. –28

2-14. In a 6-bit 1's complement binary number system, what is the decimal value represented by $(010100)_{1s}$?
a. –11 b. 43 c. –43 d. 20 e. –20

2-15. What is the range of values that can be represented in a 5-bit 2's complement binary system?
a. 0 to 31 b. –8 to 7 c. –8 to 8 d. –15 to 15 e. –16 to 15

2-16. In a 4-bit 2's complement scheme, what is the result of this operation: $(1011)_{2s} + (1001)_{2s}$?
a. 4 b. 5 c. 20 d. –12 e. overflow

2-17. Assuming a 6-bit 2's complement system, perform the following subtraction operations by converting it into addition operations:
a. $(011010)_{2s} – (010000)_{2s}$
b. $(011010)_{2s} – (001101)_{2s}$
c. $(000011)_{2s} – (010000)_{2s}$

2-18. Assuming a 6-bit 1's complement system, perform the following subtraction operations by converting it into addition operations:
a. $(011111)_{1s} – (010101)_{1s}$
b. $(001010)_{1s} – (101101)_{1s}$
c. $(100000)_{1s} – (010011)_{1s}$

2-19. Which of the following values cannot be represented accurately in the 8-bit sign-and-magnitude fixed-point number format shown in Figure 2.4?
a. 4 b. –29.5 c. 20.2 d. –3.75 e. 12.25

2-20. What does 1 110 1001 represent in this floating-point number scheme: 1-bit sign, 3-bit normalized mantissa, followed by 4-bit 2's complement exponent?
a. 0.125×2^{9} b. $–0.125 \times 2^{9}$ c. $–0.75 \times 2^{-1}$ d. $–0.75 \times 2^{-6}$ e. $–0.75 \times 2^{-7}$

2-21. How to represent $(246)_{10}$ in the following system/code?
a. 10-bit binary b. BCD c. Excess-3 d. 2421 code e. 84-2-1 code

2-22. The decimal number 573 is represented as 1111 0110 1011 in an unknown self-complementing code. Find the code for the decimal number 642.

2-23. Convert $(101011)_2$ to its corresponding Gray code value.
a. $(101011)_{Gray}$ b. $(010100)_{Gray}$ c. $(110010)_{Gray}$ d. $(111110)_{Gray}$ e. $(43)_{Gray}$

2-24. Convert $(101011)_{Gray}$ to its corresponding binary value.
a. $(101011)_2$ b. $(010100)_2$ c. $(110010)_2$ d. $(111110)_2$ e. $(010101)_2$

Answers to Quick Review Questions

2-1. (c)
2-2. (e)
2-3. (c)
2-4. (d)
2-5. (a) $(2000)_{10} = (11111010000)_2$ (b) $(0.875)_{10} = (0.111)_2$
(c) $(0.07)_{10} = (0.00010010)_2$ (d) $(12.345)_{10} = (1100.01011000)_2$
2-6. (a) $(2000)_{10} = (5555)_7$ (b) $(0.875)_{10} = (0.606061)_7$
(c) $(0.07)_{10} = (0.033003)_7$ or $(0.033004)_7$
(d) $(12.345)_{10} = (15.226223)_7$
2-7. (a) $(2000)_{10} = (3720)_8$ (b) $(0.875)_{10} = (0.7)_8$
(c) $(0.07)_{10} = (0.0437)_8$ (d) $(12.345)_{10} = (14.2605)_8$
2-8. (a) $(2000)_{10} = (7D0)_{16}$ (b) $(0.875)_{10} = (0.E)_{16}$
(c) $(0.07)_{10} = (0.12)_{16}$ (d) $(12.345)_{10} = (C.58)_{16}$
2-9. (e)
2-10. (a) $(1031)_4$ (b) $(115)_8$ (c) $(77)_{10}$ (d) $(4D)_{16}$
2-11. (b)
2-12. (a) $(101001011)_2$ (b) $(01100101)_2$ (c) $(10110111)_2$
2-13. (e)
2-14. (d)
2-15. (e)
2-16. (e)
2-17. (a) $(001010)_{2s} = (10)_{10}$ (b) $(001101)_{2s} = (13)_{10}$ (c) $(110011)_{2s} = -(13)_{10}$
2-18. (a) $(001010)_{1s} = (10)_{10}$ (b) $(011100)_{1s} = (28)_{10}$ (c) overflow
2-19. (c)
2-20. (e)
2-21. (a) $(0011110110)_2$ (b) $(0010\ 0100\ 0110)_{BCD}$ (c) $(0101\ 0111\ 1001)_{Excess\text{-}3}$
(d) $(0010\ 0100\ 1100)_{2421}$ (e) $(0110\ 0100\ 1010)_{84\text{-}2\text{-}1}$
2-22. 642 = 0100 0000 1001
2-23. (d)
2-24 .(c)

SUMMARY:

- To convert binary to decimal Start by writing the column weights; then adds the weights that correspond to each **1** in the number.
- To convert decimal to any number system, divide by base of that number system and reminders from bottom to top will be answer.
- To convert decimal fraction to any number system, multiply by base of that number system and carry from top to bottom will be answer.
- Binary to hex/octal can be converted by grouping 4/3 bits and writing equivalent hex/octal digits.
- Hex/octal to binary can be converted by writing 4/3 bit binary value corresponding to each digit.
- The 1's complement of a binary number is just the inverse of the digits
- The **2**'s complement of a binary number is **1**'s complement+1. It represent negative number in signed arithmetic i.e. 2's compliment of +8 in binary gives -8.
- In signed number MSB is 0 for positive number and 1 for negative number
- Floating point numbers

S Sign bit	E (8 bits) Biased exponent (+127)	F (23 bits) Magnitude

- Binary-to-Gray conversion: Retain the MSB. From left to right, add each adjacent pair of binary code bits to get the next Gray code bit, discarding the carry.
- Gray-to-Binary Conversion: Retain the MSB. From left to right, add each binary code bit generated to the Gray code bit in the next position, discarding the carry.
- *Excess-3 code* uses a bias value of three. i.e. *Excess-3* = BCD+0011

TIPS:

Number system conversions and arithmetic's can be done with scientific calculators and they are rarely asked in GATE. Concentrate more on 1's and 2's compliments, Binary codes, and number systems with base other than 2, 8 and 16

Simple way to do signed arithmetic is discard sign (MSB) convert rest of bit in decimal (you can use calculator for this) now apply +/- sign according to sign bit. now perform require arithmetic calculation and covert answer in binary and at the end replace sign by sign bit

GATE QUESTIONS:

Q.1

The 2's complement representation of –17 is

(a) 101110 (b) 101111

(c) 111110 (d) 110001

(b)

17 = 010001

–17 = 101111 (2's complement)

Q.2

4-bit 2's complement representation of a decimal number is 1000. The number is

(a) +8 (b) 0

(c) –7 (d) –8

Ans: D

2’s compliment represent negative number in signed arithmetic i.e. 2’s compliment of +8 in binary gives -8.